KEY

7th -8th Grade

Regional Math Meet Tests

2018

- **Individual Problems**
    - Event 1: Problem Solving (No Calculator)
    - Event 2: Problem Solving (With Calculator)
    - Event 3: Mathematical Reasoning
    - Event 4: Mental Math

- **Team Problems**
    - Event 5: Team Problems

- **Tie Breaker Question**

Name:_____   School Team: _____

Event 1: Computations Without Calculator- 20 points total

Circle your final answer!

Part I (2 points each)
Give all answers in simplest form.

1. Find $\frac{1}{3}$ of 410% of 18

$$\frac{1}{3}\left(\frac{410}{100}\right)\left(\frac{18}{1}\right) = \frac{41}{10}\left(\frac{6}{1}\right) = \frac{246}{10} = \boxed{24.6 \text{ or } 24\frac{3}{5} \text{ or } \frac{128}{5}}$$

2. If $3x + 1 = 2x$, Find $-5x^2 - 2x$

$$\text{So } x = -1 \qquad -5(-1)^2 - 2(-1) = -5(1) + 2 = \boxed{-3}$$

3. $\frac{1}{8} + \frac{3}{4}\left[\frac{1}{8} + \frac{1}{2}\right] - \frac{1}{4} = \frac{1}{8} + \frac{3}{4}\left[\frac{5}{8}\right] - \frac{1}{4} = \frac{1}{8} + \frac{15}{32} - \frac{1}{4}$

$$\downarrow$$
$$\frac{4}{8}$$

$$= \frac{4}{32} + \frac{15}{32} - \frac{8}{32} = \boxed{\frac{11}{32}}$$

4. $\left(\frac{5}{9} \div \frac{5}{6}\right) - \left(\frac{4}{5} \times \frac{3}{4}\right) = \left(\frac{5}{9} \cdot \frac{6}{5}\right) - \left(\frac{3}{5}\right) = \frac{2}{3} - \frac{3}{5} = \frac{10}{15} - \frac{9}{15} = \boxed{\frac{1}{15}}$

5. Solve for x: $4(x - 5) + 2 = 3(x - 1)$

$$4x - 20 + 2 = 3x - 3$$
$$4x - 18 = 3x - 3$$
$$\boxed{x = 15}$$

Name:_____   School Team: _____

Circle your final
answer!

Event 1: Computations Without Calculator
   Part II (2 points each)
   Give all answers in simplest form.

1. Write $\frac{3}{1500}$ as a percent.

$$\frac{3}{1500} = \frac{1}{500} \qquad \frac{1}{100} \text{ is } 1\%, \text{ So this is } \frac{1}{5} \text{ of } 1\%,$$

or $\boxed{0.2\%}$

2. What percent of 18 is 99?

$$\frac{x}{100} \cdot \frac{18}{1} = \frac{99}{1} \rightarrow \frac{18x}{100} \cdot \frac{1}{1} = \frac{99}{1} \text{ So } \boxed{550\%}$$

3. Write 0.01% as a decimal.

$$\boxed{0.0001}$$

4. $6^2 + 24 - 4 \times 3 + 2^2 \div 2 = 36 + 24 - 4 \times 3 + 4 \div 2$

$$= 36 + 24 - 12 + 2$$

$$= \boxed{50}$$

5. $\dfrac{(5-3)^2}{(3-5)^2} + \dfrac{(-7+4)^2}{(-3-6)^2} = \dfrac{2^2}{(-2)^2} + \dfrac{(-3)^2}{(-9)^2} = \dfrac{4}{4} + \dfrac{9}{81} = 1 + \dfrac{1}{9} = \boxed{1\dfrac{1}{9} \text{ or } \dfrac{10}{9}}$

Name:_____ School Team: _____

Circle your final answer!

Event 2: Computations With Calculator- 25 points total
Consumer Math (5 points each)

1. A 100-square-foot box of plastic wrap costs $1.29 while a 200-square-foot box costs $2.19. If each box has an extra 100 square feet added free, which is the better buy?

200 sq ft → $1.29 → 1.550387597 sq ft | cent

300 sq ft → $2.19 → 1.369863014 sq ft | cent

(100 sq ft for $1.29)

2. A store purchases 25 brooms for $13.42 each. It then marks up this price by 60% in order to make a profit. After selling 21 of the brooms, the store offers a 25% discount on those remaining. Find the total profit the store can expect. Round to the nearest cent.

($179.77 total profit)

21 broom profit: 160% of 13.42 is $21.47, so $8.05|broom, so $169.05 profit

other 4: 25% of $21.47 is $5.37 discount, sell for $16.10, so $2.68|broom, so $10.72 profit

3. Ian sold two boats at $2400 each. He made a profit of 20% on one boat but a loss of 20% on the other boat. Find the net profit or loss. Be sure to label if it is a profit or loss.

Boat1: $2400 is 20% profit, so   120% → $2400
                                  20% → $400
                                  100% → $2000

        $400 profit

Boat 2: $2400 is 20% loss, so   80% → $2400
                                 20% → $600
                                 100% → $3000

        $600 loss

(Overall $200 loss)

4. Suppose that you want to purchase a bike for $894. After a 7.5% local sales tax is added the cashier enters a 18% off discount code. Find the final price of the bike. Round to the nearest cent.

$894 + 0.075($894) = $961.05

$961.05 - 0.18($961.05) = ($788.06)

5. A jacket normally costs $135. It's on sale for $60.75. Find the percent decrease of the sale price from the regular price.

$$\frac{\$60.75}{\$135} = 0.45,$$ so sale cost is 45% of original cost, so
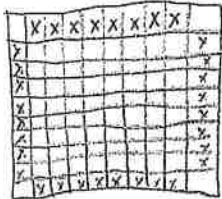
(55% decrease)

Name:_____   School Team: _____

Event 3: Mathematical Reasoning With Calculator- 35 points total

Circle your final answer!

Geometry (7 points each)
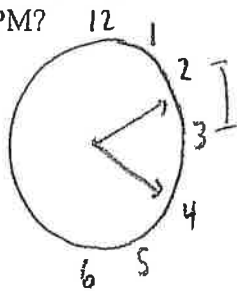_Remember to use labels when appropriate_

1. A 10 by 10 square of cubes is built (only 1 block high), glued together, and suspended in the air. The entire group of blocks is then painted. What percent of the blocks will have three faces painted?

$\frac{32}{100} = 32\%$

32%

2. How many degrees are in the acute angle formed by the hands of an analog clock at 2:20PM?

$360° \div 12 = 30°$

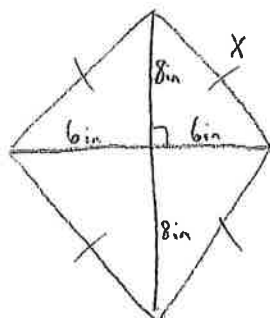2 sections of $30° \rightarrow 60°$

but the hour hand moved $\frac{20}{60}$ of way between 2+3

$\frac{1}{3} \cdot 30° = 10°$, so $60° - 10° = 50°$

$50°$

3. A rhombus has diagonals of lengths 12in and 16in. What is the perimeter of the rhombus?

$6^2 + 8^2 = x^2$

$36 + 64 = x^2$

$100 = x^2$

$10 = x$

$4 \cdot 10 = 40$

$40in$

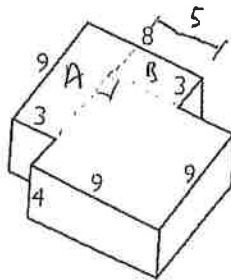Name:_____ School Team: _____

Event 3: Mathematical Reasoning With Calculator- 35 points total

Circle your final answer!

Geometry Part II (7 points each)
 _Remember to use labels when appropriate_

4. Find the volume of the figure below. All angles that appear to be right angles are right angles. All measurements are in centimeters.

$(9 \times 3 \times 4)_{cm^3} + (5 \times 3 \times 4)_{cm^3} + (9 \times 9 \times 4)_{cm^3} = \boxed{492 cm^3}$

A   B

5   8
9 A   B 3
3
9   9
4

5. Three rectangles are lined up as you see below. The first rectangle has a width on 1in and a length of 2in. The second rectangle has a width of 2in and a length of 4in. The third rectangle has a width of 4in and a length of 8in. Find the area of the shaded region.

4

2   4   8
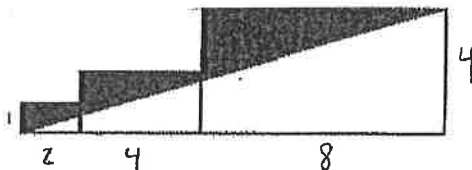
Area of $\frac{1}{2}$ lg $\square$ → $\frac{1}{2}(14_{in})(4_{in}) = 28_{in^2}$

(If lg △ shaded)

Area of 3 rect. Combined:
  $2_{in^2} + 8_{in^2} + 32_{in^2} = 42_{in^2}$

Area of shaded region:
  $42_{in^2} - 28_{in^2} = \boxed{14_{in^2}}$

Name:_____   School Team: _____

Event 4: Mental Math (no calculator)- 20 points total
(2 points each)

Example: _____23_____

1) _____332_____

2) _____264_____

3) _____164_____

4) _____-7_____

5) _____21_____

6) _____297_____

7) _____-116_____

8) _____12_____

9) _____130_____

10) _____56_____

Name:_____   School Team: _____

Circle your final answer!

Event 5: Team Problems (with calculator)- 100 points total
   Part 1: Number Theory (5 points each)

1. Some aliens have entered your classroom. They work math problems differently than what you're used to. They refuse to use any number above 4. They show you a simple pentagon that they use to solve problems, seen below. For example, to find $3 + 4$ they would start at the 0, move 3 spaces, then move 4 more spaces ending on 2. Thus $3 + 4 = 2$ to them. To find 2×3, they start at the 0 then move 3 spaces twice, ending on the 1. Thus $2 \times 3 = 1$.
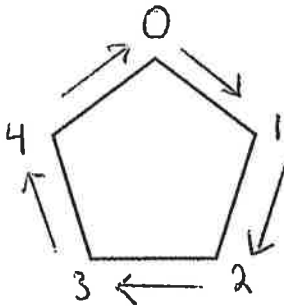
Solve each of the problems below, finding the correct number for each blank.

Notice that a multiple of 5 will always end at 0.

a) $2(3 + \boxed{2}) = 0$

$2(what) = 5x$
    $\searrow 5$

b) $(3)(\boxed{4}) + 4 = 1$ → go backwards 4 from 1, get 2
                            → $3(what) = 5x + 2$
                                could be 12, $3 \times 4 = 12$

c) $4(2) + 3(4) = \boxed{0}$
    $8 + 12 = 20$ ← is a multiple of 5

d) $(2)(3) + (4)(4) + \boxed{3} = 0$
    $6 + 16 + \underline{\quad} = 0$
    $22 + \underline{\quad} = 0$
              ↑ Think multiple of 5, could be 25
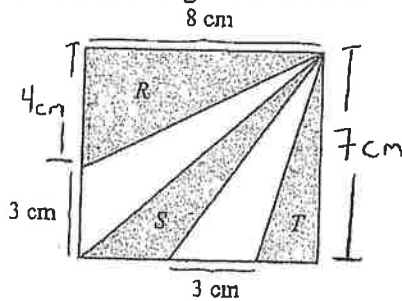
Name:_____ School Team: _____

Circle your final answer!

Event 5: Team Problems (with calculator)- 100 points total
    Part 2: Geometry and Measurement (5 points each)

Remember to use clear labels as needed!
  2. Solve each of the problems below.
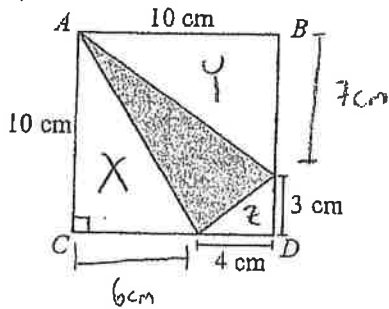
    a) The area of Region R is 16cm². Find the combined area of Regions S and T.

· Notice that the bases of S + T is 5cm. They have the same height of 7cm.
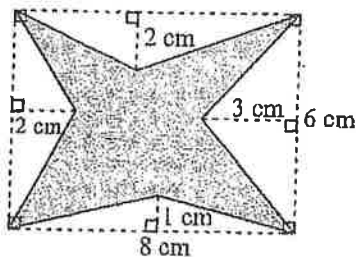
$$\frac{1}{2}(5cm)(7cm) = \boxed{17.5 cm^2}$$

    b) Find the area of the shaded region.

$$100cm^2 - 6cm^2 - 35cm^2 - 30cm^2 = \boxed{29cm^2}$$
$$\qquad\qquad\uparrow\qquad\quad\uparrow\qquad\quad\uparrow$$
$$\qquad\qquad\Delta z\qquad\quad\Delta y\qquad\quad\Delta x$$

    c) Find the area of the shaded region.

- Find area of lg rectangle, subtract 4 unshaded △ areas

$$48cm^2 - 8cm^2 - 9cm^2 - 4cm^2 - 6cm^2 = \boxed{21cm^2}$$

    d) If the length of a rectangle is increased by 20% and the width of the same rectangle is increased by 10%, by what percent will the area of the rectangle increase?

$$Area(rect) = \ell w$$
$$Area(rect_{new}) = 1.2\ell(1.1)w = 1.32\ell w$$
$$\text{Increase of } \boxed{32\%}$$
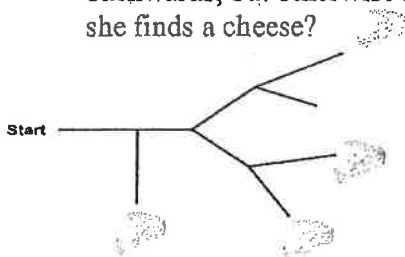
Name: _____ School Team: _____

Circle your final answer!

Event 5: Team Problems (with calculator)- 100 points total
      Part 3: Probability (6 points each)
Remember to simplify all fractions!

a) Suppose that a rat is sent into the below maze at the 'start.' If the rat cannot go backwards, but otherwise makes all the decisions at random, what is the probability that she finds a cheese?
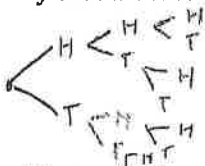
Notice that the probability of $\underline{not}$ getting a cheese is

$$\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8} \quad \text{So} \quad 1-\frac{1}{8} = \boxed{\frac{7}{8}}$$

b) Two fair dice are rolled. Find the probability that the sum of the dice is a prime number.

$$\frac{15}{36} = \boxed{\frac{5}{12}}$$

c) Three fair coins are tossed. If you know that one of the coins shows heads, what is the probability that all three coins show heads?

HTT   HTH
THT   HHT
THH   HHH
HTT

$$\boxed{\frac{1}{7}}$$

d) Suppose I have 4 slips of paper. I write one number on each slip: 1, 8, 9, and 9. I then put all four slips in a bag, mix them up, and pull them out one by one (without replacing them) to create a 4-digit number using the numbers in the order drawn. What is the probability that I will create a prime number by doing this?

$$\boxed{0}$$

$1+8+9+9 = 27$, which is divisible by 3, So all 4-digit numbers created are divisible by 3.

e) You flip a fair coin and then roll a fair die. What is the probability that you get a head and then a multiple of three?
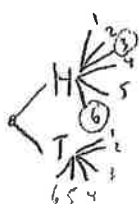
$$\frac{2}{12} = \boxed{\frac{1}{6}}$$

Name:_____ School Team:_____

Circle your final answer!

Event 5: Team Problems (with calculator)- 100 points total
Part 4: Patterns and Number Theory (6 points each)

a) A prime number p is called a Sophie Germain prime if $2p+1$ is also prime. What percent of the first ten prime numbers are Sophie Germain prime numbers?

$2 \to 5$ $7 \to 15$ $17 \to 35$ $29 \to 59$
$3 \to 7$ $11 \to 23$ $19 \to 39$
$5 \to 11$ $13 \to 27$ $23 \to 47$ $\frac{6}{10} \to$ **60%**

b) A square of numbers is created as seen below. Find the number that should go where the X is.

```
1  2   3   4
2  5  10  17
3 10  25  52
4 17  52   X
```

Adding the number above, number left, and number diagonally left gives each number in the square

$25 + 52 + 52 =$ **129**

c) A mathematician writes the words "MATH MEET" together with no spaces, repeating them infinitely as shown below. What letter will be the 2018th letter in the sequence?

MATHMEETMATHMEETMATHMEETMATHMEETMATHMEET...

8 letters repeat

$252 \times 8 = 2016$ so M is 2017, A is 2018   **A**

d) Give the next three terms in the sequence below:

1.85, 2.855, 3.91, 4.915, 5.92...

notice a difference of 1.005 between all terms   **6.925, 7.93, 8.935**

e) Give the next three terms in the sequence below:

5, 7, 12, 19, 31, 50...

Notice each is the sum of the previous 2 terms

81, 131, 212

Name:_____ School Team: _____

Circle your final answer!

TIE BREAKER (with calculator)

1) The sum of three different whole numbers is 10. Their product is 20. Find the median of the three numbers.

$$20 = 4 \times 5 = 2 \times 2 \times 5$$

$$1 + 4 + 5 = 10$$

(4)

2) The mean of three numbers a, b, and c is 7. The mean of five numbers a, b, c, d, and e is 13. Find the mean of d and e.

(22)

$$a + b + c = 21$$

$$\frac{a + b + c + d + e}{5} = 13$$

$$a + b + c + d + e = 65$$

$$d + e = 65 - 21$$

$$d + e = 44$$

$$\frac{44}{2} = 22$$

3) Ann needs a mean of 80 on her five exams in order to earn a B in her class. Her exam scores so far are 78, 90, 64 and 83. What does she need to get on her fifth exam to earn the B?

(85)

$$\frac{78 + 90 + 64 + 83 + x}{5} = 80$$

$$x = 400 - 78 - 90 - 64 - 83$$

$$x = 85$$