5/6 Mega Math Meet
2006

UW Math Meet – Mental Math Solutions

Name ANSWER KEY School Team ____________

Each answer is worth 1 pt each.

1) 55

2) 30

3) 1111

4) 24

5) 63

6) 30

7) 3412

8) 36

9) 21

10) 3

Name: Team: Answer Key

Spring 2006 UW Mega Math Meet

Individual Event II: Applications of Algebra.
Calculators allowed.

Toll House Recipe : Makes 24 large cookies

2 1/4 cups flour	3/4 cups brown sugar
1 teaspoon baking soda	1 teaspoon vanilla extract
1 teaspoon salt	2 eggs
1 cup = 2 sticks butter	2 cups = 12 ounces chocolate chips
3/4 cups white sugar	1 cup chopped pecans

1. For each question, use the above recipe to determine how much of each ingredient you need (one point each).

   (a) If you wanted to make 12 cookies, how many teaspoons of baking soda would you need?

   $\boxed{1/2}$ teaspoons

   (b) If you wanted to make 36 cookies, how many eggs would you need?

   $\boxed{3}$ eggs

   (c) If you wanted to make 40 cookies, how many cups of white sugar would you need?

   $\boxed{1\ 1/4}$ cups

   (d) If you wanted to make 68 cookies, how many cups of brown sugar would you need?

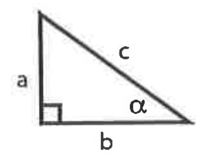
   $\boxed{2\ 1/8}$ cups

Name: Team: Answer Key

Spring 2006 UW Mega Math Meet

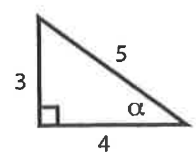
Individual Event III: Geometry and Measurement.

Calculators allowed.



We can define $sin(\alpha)$ and $cos(\alpha)$ for the above triangle as $sin(\alpha) = \frac{a}{c}$ and $cos(\alpha) = \frac{b}{c}$. For example, if we have $a = 5$ and $c = 10$, then $sin(\alpha) = \frac{1}{2}$.

1. Compute the following (one point each):

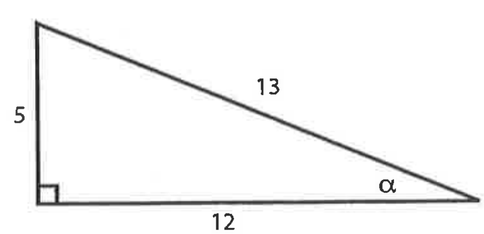


For a $(3, 4, 5)$ triangle

(a) $sin(\alpha) =$ $\boxed{3/5}$

(b) $cos(\alpha) =$ $\boxed{4/5}$

(c) $(sin(\alpha))^2 + (cos(\alpha))^2 =$ $\boxed{1}$



For a $(5, 12, 13)$ triangle

(d) $(sin(\alpha))^2 + (cos(\alpha))^2 =$ $\boxed{1}$

Name:　　　　　　Team: Answer Key

Spring 2006 UW Mega Math Meet

Individual Event IV: Problem Solving.
No Calculators.

1. Computation (one point each)

(a) $(4+3)^2 - 2 \times 7 =$ $\boxed{35}$

(b) $3\frac{1}{2} + 2\frac{3}{4} - \frac{1}{5} =$ $\boxed{6\ 1/20}$

(c) $4 \div 15 = 52 \div$ $\boxed{195}$

(d) $\sqrt{16} + 4 = 7 \times$ $\boxed{8/7}$

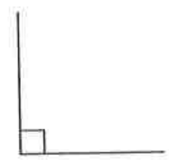
(e) 5 percent of $\boxed{600}$ $= 30$

Angles Everywhere

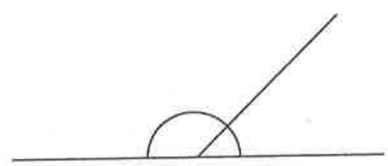
Answer Key

In this series of problems we'll be finding the angle measurements of various angles without needing to use a protactor. An angle measurement runs between 0 and 360 degrees. You'll first need some basic facts about angles.

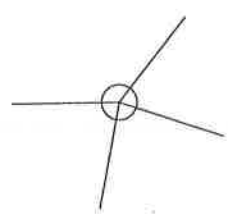
A right angle (like the one at a corner of a piece of paper) has a measurement of 90 degrees.



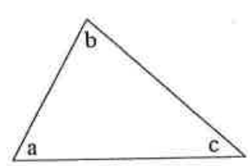
Two angles that form a straight line are called supplementary angles, and together their measurements add to 180 degrees.



If angles together completely encircle a point, together their measurements add up to 360 degrees.



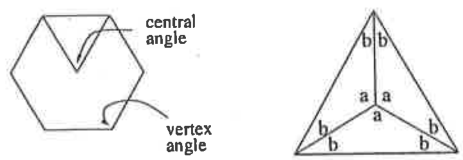
The angles of a triangle always add up to 180 degrees.



$a + b + c = 180$

For the next set of problems, we'll be working with regular polygons. A *regular polygon* is a shape made of straight lines so that all its edges have the same length and all its angles have the same measurement. For example, a square is a regular polygon with four sides. It's regular since all its sides have the same length and all its angles are 90 degrees.

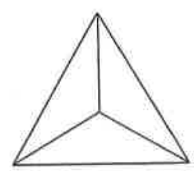
One of the angles of a regular polygon is called a *vertex angle*. If we draw lines from each vertex to the center of the polygon, we form congruent triangles. The angles around the center point all have the same measurement, and are called *central angles*. Those same lines divide the vertex angles into two equal parts.



Find the central angle and vertex angle of the following regular polygons.

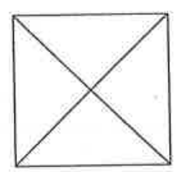
Regular Triangle (3 Sides)

(1 Point) central angle = 120 degrees
(3 Points) vertex angle = 60 degrees



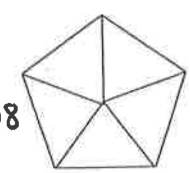
Regular Quadrilateral (4 Sides)

(1 Point) central angle = 90 degrees
(3 Points) vertex angle = ~~45~~ degrees 90



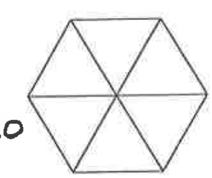
Regular Pentagon (5 Sides)

(1 Point) central angle = 72 degrees
(3 Points) vertex angle = ~~54~~ degrees 108



Regular Hexagon (6 Sides)

(1 Point) central angle = 60 degrees
(3 Points) vertex angle = ~~60~~ degrees 120



Regular 100-gon (100 Sides)

(3 Points) central angle = 3.6 degrees
(5 Points) vertex angle = ~~88.2~~ degrees 176.4

Regular 1000-gon (1000 Sides)

(3 Points) central angle = .36 degrees
(5 Points) vertex angle = ~~89.82~~ degrees 179.64

Answer Key

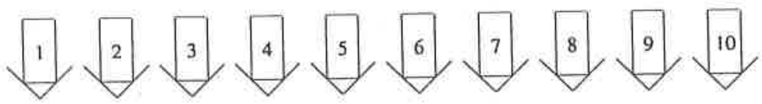
Light Switches: On and Off...

In this series of problems, we'll be looking at light switches that get flipped ON or OFF. Each light switch will have a number. For example, we'll use the picture on the left to denote the first light switch flipped ON and the picture on the right to denote the first light switch flipped OFF.

ON 

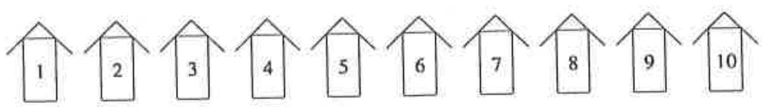
OFF

Suppose that there is a row of ten light switches, all starting turned OFF.

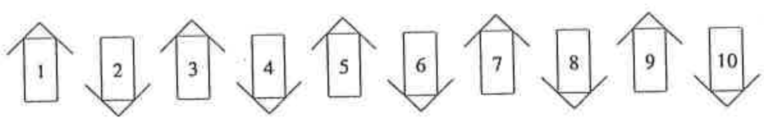


Ten people walk by the switches, flipping some of them as follows.

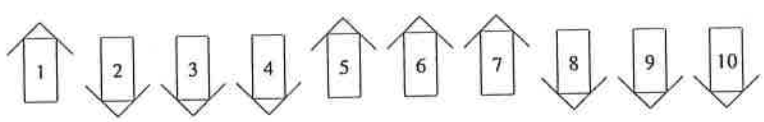
- The *first* person walks along the row and flips *every* switch, resulting in the following:



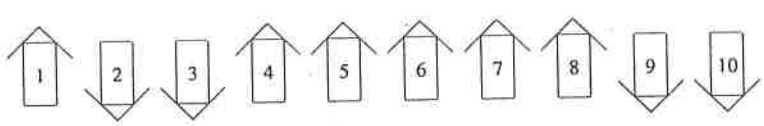
- The *second* person then walks along the row and flips every *second* switch, resulting in the following:



- The *third* person then walks along the row and flips every *third* switch, resulting in the following:



- The *fourth* person then walks along the row and flips every *fourth* switch, resulting in the following:



Ten Switches Continued...

(2 Points) Which people flipped switch #7?

Persons #1 and #7.

(2 Points) Which people flipped switch #8?

Persons #1, #2, #4, and #8.

(2 Points) Which people flipped switch #9?

Persons #1, #3, and #9.

(2 Points) Which people flipped switch #10?

Persons #1, #2, #5, and #10.

(5 Points) Which light switches were ON at the end?

Lights #1, #4, and #9.

Answer Key

The Game of NIM

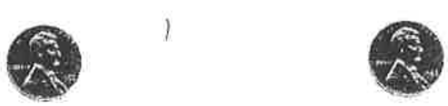
In the game of NIM, two players take turns removing coins from a collection of piles. For example, the game may start with a pile of six pennies, a pile of five pennies, and a pile of four pennies as pictured.

On their turn, each player removes some number of pennies from one of the piles. As an example, the first player could remove two pennies from the middle pile on their turn, leaving the following.

Continuing the example, the second player could then remove all six pennies from the left pile on their turn, leaving the following.

The winner of NIM is the player who takes the very last penny.

Let's quickly look at a very small game of NIM consisting of two piles with one penny each.



Player One

Player Two

The first player has two options: either remove the only penny from the pile on the left or remove the only penny from the pile on the right. Either way, the second player will win by taking the remaining penny from the other pile. We therefore circled *Player Two* on the right to indicate that the second player will win.

Two Pile NIM

When there are two piles, the game of NIM is slightly more complicated. For example, Player One should win the following setup.

Player 1

Player One will start by removing one penny from the pile on the left, leaving the following.

Player 2 wins.

Player Two will then have to remove one penny, either from the pile on the left or the pile on the right. Either way, Player One will be left with a single penny in a single pile, which they will take to win.

Note that Player One could foolishly take both pennies from the pile on the left, which would allow Player Two to win by removing the last penny from the pile on the right. But Player One can't lose unless they make a mistake like this. So we say that Player One should win.

For each game below pictured below, who should win? Circle who should win for each game.

(4 Points) Game One

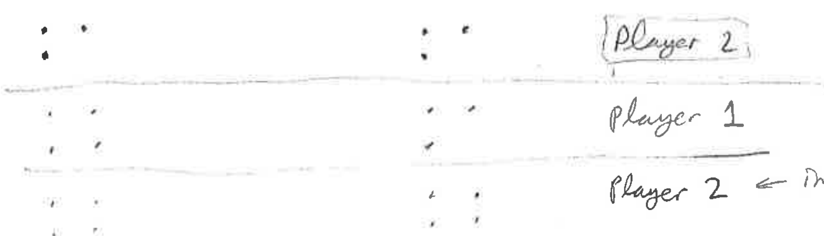
Player One

Player Two

(4 Points) Game Two

Player One

Player Two



Player 2

Player 1

Player 2 ← mistakes

Three Pile Nim

When there are three piles, the game of Nim gets significantly more complicated.

For each game below pictured below, who should win? Circle who should win for each game.

(6 Points) Game One

Player One

Player Two

(6 Points) Game Two

Player One

Player Two

(8 Points) Game Three

Player One

Player Two

(6 Points) Game Four

In game four, Player One wins.

Player One

Player Two

What does Player One do on their first turn to make sure they win?

REMOVE THE (ONLY) PENNY IN THE RIGHT PILE.

SCORE
OUT OF 70

Name: Team:

Spring 2006 UW Mega Math Meet

Individual Event II: Applications of Algebra.
Calculators allowed.

Toll House Recipe : Makes 24 large cookies

2 1/4 cups flour	3/4 cups brown sugar
1 teaspoon baking soda	1 teaspoon vanilla extract
1 teaspoon salt	2 eggs
1 cup = 2 sticks butter	2 cups = 12 ounces chocolate chips
3/4 cups white sugar	1 cup chopped pecans

1. For each question, use the above recipe to determine how much of each ingredient you need (one point each).

   (a) If you wanted to make 12 cookies, how many teaspoons of baking soda would you need?

   $\boxed{}$ teaspoons

   (b) If you wanted to make 36 cookies, how many eggs would you need?

   $\boxed{}$ eggs

   (c) If you wanted to make 40 cookies, how many cups of white sugar would you need?

   $\boxed{}$ cups

   (d) If you wanted to make 68 cookies, how many cups of brown sugar would you need?

   $\boxed{}$ cups

(e) If you wanted to make 99 cookies, how many ounces of chocolate chips would you need?

$\boxed{}$ ounces

2. For each question, given the following ingredients (and a lot of everything else), what is the maximum number of cookies you can make? (One point for a, two points each for b and c.)

   (a) (1 pt.) How many cookies can you make with 3 sticks of butter, 2 cups of brown sugar, and a lot of the other ingredients?

   $\boxed{}$ cookies

   (b) (2 pts.) How many cookies can you make with 5 eggs, 3 1/2 cups of flour, 1 1/3 cups chopped pecans, and a lot of the other ingredients?

   $\boxed{}$ cookies

   (c) (2 pts.) How many cookies can you make with 5 1/2 sticks of butter, 4 cups of white sugar, 6 3/4 cups of flour, and a lot of the other ingredients?

   $\boxed{}$ cookies

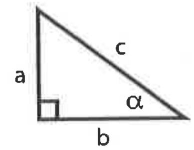
Total: ______________ (out of 10 points)

Name: Team:

Spring 2006 UW Mega Math Meet

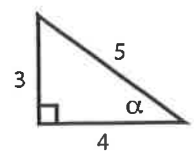
Individual Event III: Geometry and Measurement.

Calculators allowed.



We can define $sin(\alpha)$ and $cos(\alpha)$ for the above triangle as $sin(\alpha) = \frac{a}{c}$ and $cos(\alpha) = \frac{b}{c}$. For example, if we have $a = 5$ and $c = 10$, then $sin(\alpha) = \frac{1}{2}$.

1. Compute the following (one point each):

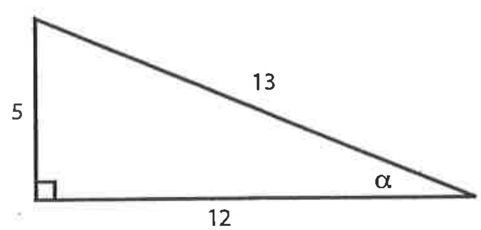


For a $(3, 4, 5)$ triangle

(a) $sin(\alpha) =$ $\boxed{}$

(b) $cos(\alpha) =$ $\boxed{}$

(c) $(sin(\alpha))^2 + (cos(\alpha))^2 =$ $\boxed{}$



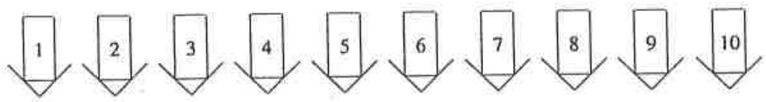
For a $(5, 12, 13)$ triangle

(d) $(sin(\alpha))^2 + (cos(\alpha))^2 =$ $\boxed{}$

Light Switches: On and Off...

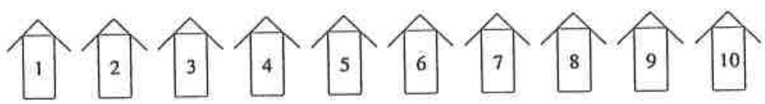
In this series of problems, we'll be looking at light switches that get flipped ON or OFF. Each light switch will have a number. For example, we'll use the picture on the left to denote the first light switch flipped ON and the picture on the right to denote the first light switch flipped OFF.

Suppose that there is a row of ten light switches, all starting turned OFF.

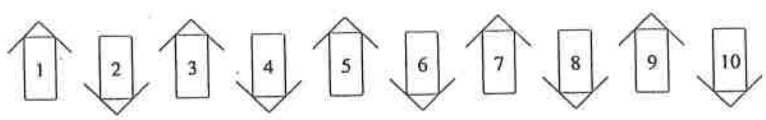


Ten people walk by the switches, flipping some of them as follows.

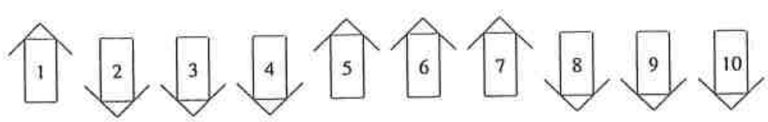
- The *first* person walks along the row and flips *every* switch, resulting in the following:



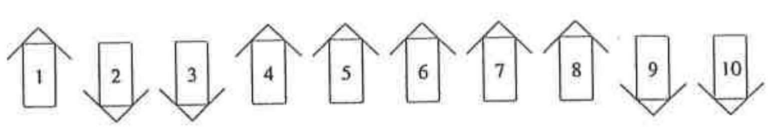
- The *second* person then walks along the row and flips every *second* switch, resulting in the following:



- The *third* person then walks along the row and flips every *third* switch, resulting in the following:



- The *fourth* person then walks along the row and flips every *fourth* switch, resulting in the following:



Ten Switches Continued...

The remaining people then walk along the row and continue the pattern of flipping switches. Indicate which switches are turned ON and which are turned OFF to the following questions by adding arrows to each switch. completing the following questions.

(2 Points) If the *fifth* person then walks along the row and flips every *fifth* switch, draw the switches after the fifth person.

1	2	3	4	5	6	7	8	9	10

(2 Points) If the *sixth* person then walks along the row and flips every *sixth* switch, draw the switches after the sixth person.

1	2	3	4	5	6	7	8	9	10

(2 Points) If the *seventh* person then walks along the row and flips every *seventh* switch, draw the switches after the seventh person.

1	2	3	4	5	6	7	8	9	10

(2 Points) If the *eighth* person then walks along the row and flips every *eighth* switch, draw the switches after the eighth person.

1	2	3	4	5	6	7	8	9	10

(2 Points) If the *ninth* person then walks along the row and flips every *ninth* switch, draw the switches after the ninth person.

1	2	3	4	5	6	7	8	9	10

(2 Points) If the *tenth* person then walks along the row and flips every *tenth* switch, draw the switches after the tenth person.

1	2	3	4	5	6	7	8	9	10

A Hundred Switches...

Now suppose that instead of having ten switches and ten people, we have one hundred switches and one hundred people. As before, the switches all start turned off. The first person walks by flipping every switch, then the second person walks by flipping every second switch, then the third person walks by flipping every third switch, and so on, until the one hundredth person walks by flipping every one hundredth switch.

(3 Points) Which switches will person #21 flip?

(3 Points) Which switches will person #45 flip?

(3 Points) Which switches will person #73 flip?

(5 Points) Which people will flip switch #3?

(5 Points) Which people will flip switch #32?

(4 Points) Which people will flip switch #10?
Will switch #10 end up ON or OFF?

(4 Points) Which people will flip switch #49?
Will switch #49 end up ON or OFF?

(4 Points) Will switch #60 end up ON or OFF?

(4 Points) Will switch #100 end up ON or OFF?

(10 Points) Which switches will be turned ON after all one hundred people walk by?

SCORE
OUT OF 70

The Game of NIM

In the game of NIM, two players take turns removing coins from a collection of piles. For example, the game may start with a pile of six pennies, a pile of five pennies, and a pile of four pennies as pictured.

On their turn, each player removes some number of pennies from one of the piles. As an example, the first player could remove two pennies from the middle pile on their turn, leaving the following.

Continuing the example, the second player could then remove all six pennies from the left pile on their turn, leaving the following.

The winner of NIM is the player who takes the very last penny.

Let's quickly look at a very small game of NIM consisting of two piles with one penny each.

Player One

Player Two

The first player has two options: either remove the only penny from the pile on the left or remove the only penny from the pile on the right. Either way, the second player will win by taking the remaining penny from the other pile. We therefore circled *Player Two* on the right to indicate that the second player will win.

Two Pile Nim

When there are two piles, the game of Nim is slightly more complicated. For example, Player One should win the following setup.

Player One will start by removing one penny from the pile on the left, leaving the following.

Player Two will then have to remove one penny, either from the pile on the left or the pile on the right. Either way, Player One will be left with a single penny in a single pile, which they will take to win.

Note that Player One could foolishly take both pennies from the pile on the left, which would allow Player Two to win by removing the last penny from the pile on the right. But Player One can't lose unless they make a mistake like this. So we say that Player One should win.

For each game below pictured below, who should win? Circle who should win for each game.

(4 Points) Game One

Player One

Player Two

(4 Points) Game Two

Player One

Player Two

Three Pile NIM

When there are three piles, the game of NIM gets significantly more complicated.

For each game below pictured below, who should win? Circle who should win for each game.

(6 Points) Game One

Player One

Player Two

(6 Points) Game Two

Player One

Player Two

(8 Points) Game Three

Player One

Player Two

(6 Points) Game Four

In game four, Player One wins.

Player One

Player Two

What does Player One do on their first turn to make sure they win?

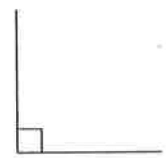
SCORE

OUT OF 70

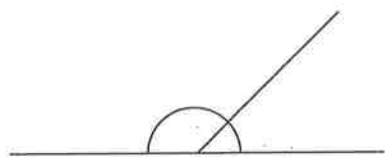
Angles Everywhere

In this series of problems we'll be finding the angle measurements of various angles without needing to use a protactor. An angle measurement runs between 0 and 360 degrees. You'll first need some basic facts about angles.

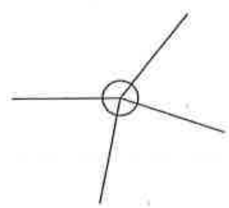
A right angle (like the one at a corner of a piece of paper) has a measurement of 90 degrees.



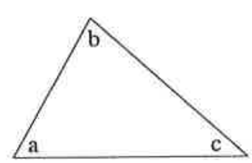
Two angles that form a straight line are called supplementary angles, and together their measurements add to 180 degrees.



If angles together completely encircle a point, together their measurements add up to 360 degrees.



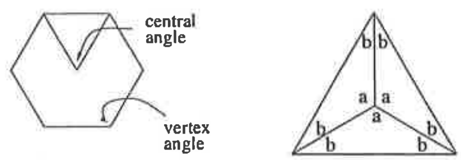
The angles of a triangle always add up to 180 degrees.



a + b + c = 180

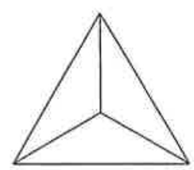
For the next set of problems, we'll be working with regular polygons. A *regular polygon* is a shape made of straight lines so that all its edges have the same length and all its angles have the same measurement. For example, a square is a regular polygon with four sides. It's regular since all its sides have the same length and all its angles are 90 degrees.

One of the angles of a regular polygon is called a *vertex angle.* If we draw lines from each vertex to the center of the polygon, we form congruent triangles. The angles around the center point all have the same measurement, and are called *central angles.* Those same lines divide the vertex angles into two equal parts.



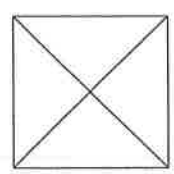
Find the central angle and vertex angle of the following regular polygons.

Regular Triangle (3 Sides)



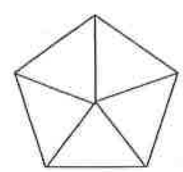
(1 Point) central angle = degrees
(3 Points) vertex angle = degrees

Regular Quadrilateral (4 Sides)



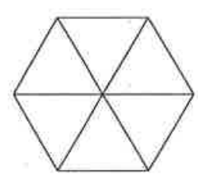
(1 Point) central angle = degrees
(3 Points) vertex angle = degrees

Regular Pentagon (5 Sides)



(1 Point) central angle = degrees
(3 Points) vertex angle = degrees

Regular Hexagon (6 Sides)



(1 Point) central angle = degrees
(3 Points) vertex angle = degrees

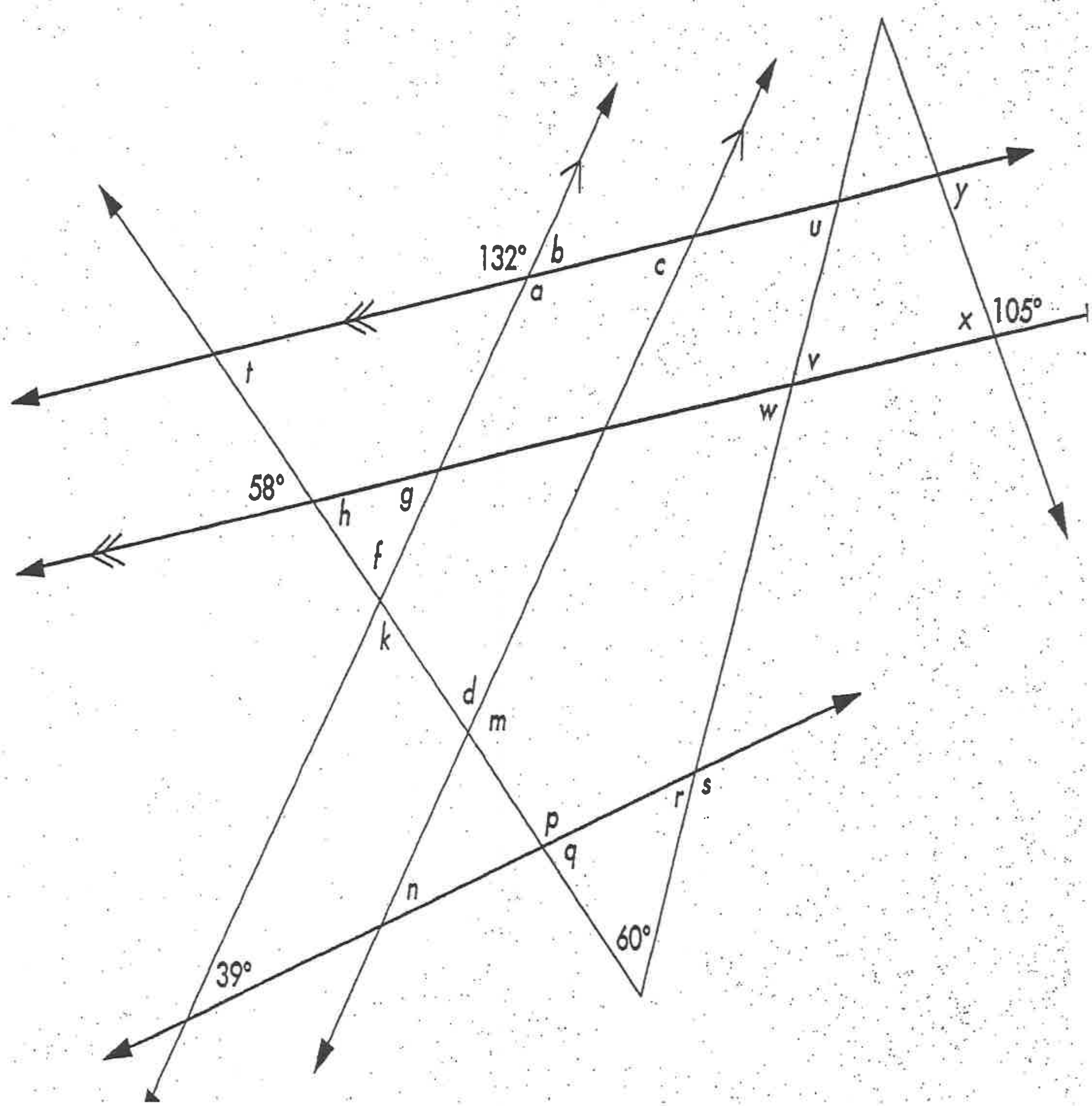
Regular 100-gon (100 Sides)

(3 Points) central angle = degrees
(5 Points) vertex angle = degrees

Regular 1000-gon (1000 Sides)

(3 Points) central angle = degrees
(5 Points) vertex angle = degrees

2.



Name: Team:

Spring 2006 UW Mega Math Meet

Individual Event IV: Problem Solving.
No Calculators.

1. Computation (one point each)

   (a) $(4+3)^2 - 2 \times 7 = \boxed{}$

   (b) $3\frac{1}{2} + 2\frac{3}{4} - \frac{1}{5} = \boxed{}$

   (c) $4 \div 15 = 52 \div \boxed{}$

   (d) $\sqrt{16} + 4 = 7 \times \boxed{}$

   (e) 5 percent of $\boxed{}$ $= 30$