7th -8th Grade

Regional Math Meet Tests

2020

- **Individual Problems**
  - Event 1: Problem Solving (No Calculator)
  - Event 2: Problem Solving (With Calculator)
  - Event 3: Mathematical Reasoning
  - Event 4: Mental Math

- **Team Problems**
  - Event 5: Team Problems

- **Tie Breaker Question**

Name:_____ School Team: _____

Event 1: Computations Without Calculator- 20 points total

Circle your final answer!

Part I (2 points each)

Give all answers in simplest form.

1. Write $\frac{3^2 \times 6^{-2}}{10^{-3} \times 5^2}$ as an integer

$\frac{9 \times \frac{1}{36}}{\frac{1}{1000} \times 25} = \frac{\frac{9}{36}}{\frac{25}{1000}} = \frac{9}{36} \cdot \frac{1000}{25} = \frac{1}{4} \cdot \frac{40}{1} = 10$

2. What percent of 1.6 is 17.2?

$1.6x = 17.2$

$$\begin{array}{r} 10.75 \\ 1.6\overline{)17.2000} \\ -16 \\ \hline 120 \\ -112 \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

1075%

3. $\left[\sqrt[3]{27} - \left(\frac{1}{2}\right)^2\right] \times \frac{2}{3} = \left[3 - \frac{1}{4}\right] \times \frac{2}{3}$

$= 2\frac{3}{4} \times \frac{2}{3}$

$= \frac{11}{4} \times \frac{2}{3} = \boxed{\frac{11}{6} \text{ or } 1\frac{5}{6}}$

4. $-5^2 - 25 \div 5 \times 2 - 7 = -25 - 5 \times 2 - 7 = -25 - 10 - 7 = -42$

5. $\frac{1 - \frac{2}{3}(6+15)}{|542 - 2(349)|}$

$\frac{2}{3}(21) = 14$

$= \frac{1 - \frac{2}{3}(21)}{|542 - 698|} = \frac{1 - 14}{156} = \frac{-13}{156} = \frac{-1}{12}$

Name:_____ School Team: _____

Circle your final answer!

Event 1: Computations Without Calculator

Part II (2 points each)

Give all answers in simplest form.

1. $0.135 \times 240 \div \frac{1}{8}$ $= 0.135 \times 240 \times 8 = 240 \times 1.08 =$ 259.2 or $259\frac{1}{5}$

$$\begin{array}{r} 24 \\ 0.135 \\ \times \quad 8 \\ \hline 1.080 \end{array} \qquad \begin{array}{r} 240 \\ \times 1.08 \\ \hline 1920 \\ 24000 \\ \hline 25920 \end{array}$$

2. Find the reciprocal of $1.73 \times 2\frac{4}{5}$

$1\frac{73}{100} \times 2\frac{4}{5} = \frac{173}{100} \times \frac{14}{5} = \frac{173}{50} \times \frac{7}{5} = \frac{1121}{250}$ $\frac{250}{1121}$

3. Find 245% of $\frac{5}{6}$ of 96

$96 \div 6 = 16$

$5 \times 16 = 80$

$2.45 \times 80 = 196$

196

4. $4\frac{3}{4} \times 6 + 103 - 72.5$ $= 24 + 3 + 1.5 + 30.5 = 59$

(4×6, $\frac{1}{2} \times 6$, $\frac{1}{4} \times 6$, $103 - 72.5$)

59

5. $(\frac{5}{6} - \frac{1}{3}) \div 1\frac{1}{3} + \frac{1}{2}$ $= \frac{1}{2} \cdot \frac{3}{4} + \frac{1}{2} = \frac{3}{8} + \frac{1}{2} = \frac{7}{8}$

$\frac{1}{3} \downarrow \frac{2}{6}$ $\frac{1}{2} \downarrow \frac{4}{8}$

$\frac{7}{8}$

Name:_____ School Team: _____

Circle your final answer!

Event 2: Computations With Calculator- 25 points total

Consumer Math (5 points each)

1. April bought five copies of a favorite book, one for herself and four as gifts. The books were on sale for 10% off their normal price. After paying 3.8% tax and 50 cents to gift wrap each book that is a gift, April had spent a total of $95.19. Find the usual cost of each book. Always round to the nearest cent.

$95.19 - $2 = $93.19 [price before gift wrap]
x is price without tax of all 5 books
$1.038x = $93.19
$x = $89.78

p is price of 5 before sale
$0.9p = $89.78
$p = $99.76

$99.76 ÷ 5 = $19.95 to find individual cost of each book

$19.95

2. A local vendor sells 3 cucumbers for $1, 2 cucumbers for 75 cents, and 1 cucumber for 40 cents. A restaurant purchased 35 cucumbers. Find the lowest price they might expect to pay for the cucumbers.

3 for $1 → 33 this way → $11
2 for 75¢ → 2 this way → $0.75

$11.75

3. A real estate agent earns a 7% commission on all home sales. Find the price of the house if a commission was $5,775.00.

x is the cost of the house
$0.07x = $5,775 $x = $82,500

$82,500

4. 150 shares of a certain stock are purchased for $12.89 a share. The broker negotiating the purchase charged a 2% commission on the sale. A week later, stock prices rose to $14.82. If you sell all the shares at that price and again must pay a 2% commission on the sale, find the net profit.

$12.89 × 150 = $1933.50, so $38.67 for the broker
$14.82 × 150 = $2223, so $44.46 for the broker
Total paid: $1933.50 + $38.67 + $44.46 = $2016.63

Profit is $2223 − $2016.63

$206.37

5. June bought a dress to wear to a wedding. The usual price was discounted by 25%. Additionally, June used a coupon to receive 10% off the sale price. The 8.3% sales tax came to $6.69. Find the usual price of the dress. Be sure to round to the nearest cent with every calculation.

x is the price after the coupon
$0.083x = $6.69
$x = $80.60

p is the sale price
$0.9p = $80.60
$p = $89.56

n is the usual price
$0.75n = $89.56
$n = $119.41

$119.41

Name:_____ School Team: _____

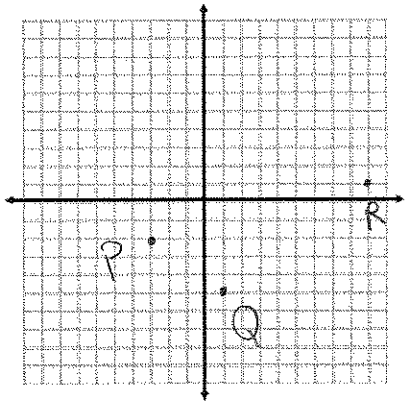
Event 3: Mathematical Reasoning With Calculator- 35 points total

Circle your final answer!

Geometry (7 points each)

Remember to use labels when appropriate

1. Three vertices of a parallelogram are P (– 3, – 2), Q (1, – 5), and R (9, 1). P is the vertex diagonal from R. Find the sum of the coordinates of the fourth vertex.



Notice from Q to R you move right 8 units and up 6 units.
Moving the same from P gives us (5, 4)

Sum: Then 5+4=9

9

2. A right triangle has side lengths of 10cm, 24cm, and 26cm. A rectangle with an area equal to the right triangle is 3cm wide. Find the perimeter of the rectangle.

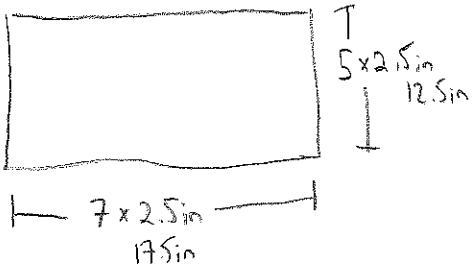
Notice 26cm must be the hypotenuse.
Then Area(Δ) = $\frac{1}{2} \cdot 10cm \cdot 24cm = 120cm^2$

$120 \div 3 = 40$, meaning the rectangle's other lengths are 40cm

$P = 3cm + 3cm + 40cm + 40cm = 86cm$

86cm

3. A rectangular box fits exactly 5 cylindrical candles on its shorter side and exactly 7 of the same candles on its longer side. Each candle has a diameter of 2.5in and a height of 4in, which matches the height of the box. If the box is filled with candles, how much empty space is there? Use 3.14 for pi.



Box volume: $4in \times 12.5in \times 17.5in = 875in^3$

Candle volume: $35 \times 4in \times 3.14(1.25in)^2 = 686.875in^3$

$875in^3 - 686.875in^3 = 188.125in^3$

$188.125in^3$

Name:_____ School Team: _____

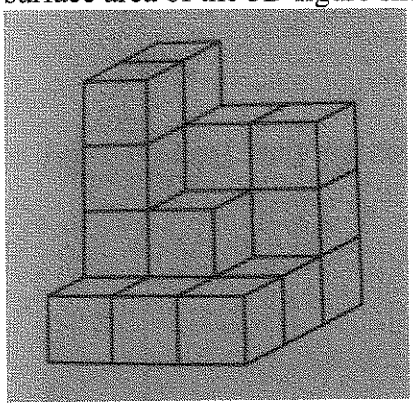
Event 3: Mathematical Reasoning With Calculator- 35 points total

Circle your final answer!

Geometry Part II (7 points each)

Remember to use labels when appropriate

4. Shameka has 20 cubes arranged as shown. Each cube has a volume of $1cm^3$. Find the surface area of the 3D figure she made.



If a cube has volume $1cm^3$, then each face is $1cm^2$

Notice 6 ways to look

From front: $10cm^2$

From top: $9cm^2$

From right: $9cm^2$

From left: $9cm^2$

From back: $10cm^2$

From bottom: $9cm^2$

Total surface area: $56cm^2$

5. A solid cube whose edge is 4cm long weighs 192g. How heavy will a similar cube be if its edge measures 9cm?

Similarity scale

$4cm \times 2\frac{1}{4} = 9cm$

$192g \times \left(2\frac{1}{4}\right)^3 = 2187g$

cube for volume

Name:_____ School Team: _____

Event 4: Mental Math (no calculator)- 20 points total
(2 points each)

Example: 23

1) 5247

2) 2400

3) −431

4) 85

5) −28

6) 446

7) 106.5 or $106\frac{1}{2}$

8) 49.5 or $49\frac{1}{2}$

9) 4694

10) 79

Name:_____ School Team: _____

Circle your final answer!

Event 5: Team Problems (with calculator)- 100 points total

Part 1: Factorials (4 points each)

In mathematics, the notation n! is read "n factorial". To find n!, we find the value of

$$1 \times 2 \times 3 \times \ldots (n-1) \times n$$

For example, $4! = 1 \times 2 \times 3 \times 4$

1. Find x if $6! \times 7! = x!$

8 is 2·4, 9 is 3·3 from 6, 10 is 2·5 from 6

$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 = 6! \cdot 7! = 10!$

8 is 2·4, 10 is 2·5, 9 is 3·3 from 6, 7!

$x = 10$

2. Find n if $(5!)(n) = (-2)(-4)(-6)(-8)(-10)$

$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5n = -3840$

$120n = -3840$

$n = 32$

$n = 32$

3. Find the value of $\frac{1000! - 999!}{999!}$

$\frac{1000! - 999!}{999!} = \frac{999!(1000-1)}{999!} = 999$

4. Find the GCF of 8! and 4^3.

$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8$

4, 4·2, another 4

There are 3 factors of 4 in 8!, or 8! is divisible by $4^3 = 64$

64

5. If we wrote out 50! As a whole number, how many zeros would be on the end of the number?

Zeros happen as multiples of 10. $10 = 5 \cdot 2$, and with a factorial we are guaranteed at least one 2 for every 5. Count the number of fives.

5 10 15 20 25 30 35 40 45 50

25 has 2, 50 has 2

12 zeros

Name:_____ School Team: _____

Circle your final answer!

Event 5: Team Problems (with calculator)- 100 points total

Part 2: Statistics (7 points each)

Remember to label as appropriate!

1. The mean April rainfall of a certain city was 3.39 inches over a 20 year period. Suppose this month it's unusually rainy. How much rainfall is required to increase the mean by a minimum of 0.11in?

$20(3.39\text{in}) = 67.8\text{in}$ total in 20yrs
$20(3.39\text{in} + 0.11\text{in}) = 70\text{in}$ total desired
$19(3.39\text{in}) = 64.41\text{in}$ in previous yrs

$70\text{in} - 64.41\text{in} = 5.59\text{in}$

5.59in

2. Using the frequency table provided, find the mean, median, and mode. Be sure to clearly label each. Round to the nearest cent.

Income	Frequency
$1,250,000	1
$345,000	3
$130,000	6
$85,000	8
$55,000	12

Mean → $1250000 + 3($345000) + 6($130000) + 8($85000) + 12($55000) =
$4,405,000
$4,405,000 ÷ 30 ≈ $146,833.33 ← Mean

Median → $85,000 (middle value)

Mode → $55,000 (most occuring)

3. The mean of a set of 50 numbers is 38. The numbers 45 and 55 are discarded from the set. Find the mean of the set of remaining numbers.

$50(38) = 1900$ ← original sum of all 50

$1900 - 45 - 55 = 1800$ ← sum of remaining

$1800 \div 48 = 37.5$

37.5

4. Annabeth wants to earn a 90% overall average in her math class. Her current scores are 83%, 92%, 78%, 96%, and 89%. What grade does she need to average on the next three assessments in order to meet her goal? Let x be what's needed to average on each of next 3 assessments

$83 + 92 + 78 + 96 + 89 + 3x = 90(8)$
$438 + 3x = 720$
$3x = 282$
$x = 94$

values are percents [except 3, 8]

94%

Name:_____ School Team: _____

Circle your final answer!

Event 5: Team Problems (with calculator)- 100 points total

Part 3: Algebra (4 points each)

Remember to simplify all fractions!

1. Find the sum of the reciprocals of two numbers, given that these numbers have a sum of 50 and a product of 25. Let x, y be the two numbers.

$x+y=50$

$xy=25$

Sum of reciprocals: $\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\frac{y+x}{xy}=\frac{50}{25}=2$

2. If $(mx+7)(5x+n)=px^2+15x+14$, find $m(n+p)$

$(mx+7)(5x+n)=5mx^2+35x+mnx+7n$

So

$5m=p$ $(35+mn)=15$ and $7n=14$

So $n=2$

Then $35+2m=15$

$2m=-20$

$m=-10$

$5(-10)=p$

$-50=p$

Now $m(n+p)=$

$-10(2+-50)=$

$-10(-48)=$

480

3. Find w in terms of x if $(x-3)(x+4)=(x+3)(x-4)+w$

$x^2+x-12=x^2-x-12+w$

$-x^2+x+12 \quad -x^2+x+12$

$2x=w$

2x

4. Find the value of $x-y$ if $x^2-y^2=10$ and $x+y=10$

$(x^2-y^2)=(x+y)(x-y)=10(x-y)=\frac{10}{10}$

$10 \quad 10$

$x-y=1$

5. Give the LCM of x^3, x^4, and x^5.

x^5

Note $x^3 \cdot x^2=x^5$

$x^4 \cdot x=x^5$

$x^5 \cdot 1=x^5$

Name:_____ School Team: _____

Circle your final answer!

Event 5: Team Problems (with calculator)- 100 points total

Part 4: Probability (8 points each)

next are 13, 21, 34, 55, 89...

Write all answers as a simplified fraction

1. The Fibonacci Sequence begins with the numbers 1, 1, 2, 3, 5, 8,… To create the next value in the sequence, you find the sum of the previous two values. A fair die is labeled with the first six numerals of this sequence- one on each face.

   a) If the die is rolled twice and the sum of the numbers is found, what is the probability that the sum is in the sequence?

   36 possible sums, marked w/ ★ if in sequence $\frac{14}{36} = \frac{7}{18}$

   b) If the die is rolled twice and the sum of the numbers is found, what is the probability that the sum is even?

   36 possible sums, marked w/ + if even $\frac{20}{36} = \frac{10}{18} = \frac{5}{9}$

   c) If the die is rolled twice and the product of the numbers is found, what is the probability that the product is in the sequence?

   Marked ★ $\frac{20}{36} = \frac{5}{9}$

   d) If the die is rolled twice and the product of the numbers is found, what is the probability that the product is at least 6?

   Marked w/ + $\frac{19}{36}$

Finding all possible Sums

	1	1	2	3	5	8
1	2+ ★	2+ ★	3 ★	4+	6+	9
1	2+ ★	2+ ★	3 ★	4+	6+	9
2	3 ★	3 ★	4+	5 ★	7	10+
3	4+	4+	5 ★	6+	8+ ★	11
5	6+	6+	7	8+ ★	10+	13 ★
8	9	9	10+	11	13 ★	16+

Finding all possible products

	1	1	2	3	5	8
1	1 ★	1 ★	2 ★	3 ★	5 ★	8+ ★
1	1 ★	1 ★	2 ★	3 ★	5 ★	8+ ★
2	2 ★	2 ★	4	6+	10+	16+
3	3 ★	3 ★	6+	9+	15+	24+
5	5 ★	5 ★	10+	15+	25+	40+
8	8+ ★	8+ ★	16+	24+	40+	64+

Name:_____ School Team: _____

Circle your final answer!

TIE BREAKER (with calculator)

1) A number is randomly selected from the numbers 1 through 60. Given that the number is prime, what is the probability that one of its digits is 9? Write your answer as a fraction in simplest terms.

Primes from 1 to 60: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59

17 possible, 3 have a 9 as a digit

$\frac{3}{17}$

2) P is the product of all prime numbers between 50 and 70. How many positive integers are factors of P?

53, 59, 61, 67 are primes from 50 - 70

16

Factor list

1	53·59·61·67	53·59	61·67
53	59·61·67	53·61	59·67
59	53·61·67	53·67	59·61
61	53·59·67		
67	53·59·61		

3) How many primes have the property that when 3 is added to the cube of the prime, the result is another prime number?

let p be prime

$p^3 + 3$ → will it be prime?

Note that p is odd, unless $p = 2$. If we find p^3 and p is odd, then p^3 is odd. The sum of 2 odd numbers $(p^3 + 3)$ is even. But 2 is the only even prime, and so $p^3 + 3$ can't be prime if p is odd.

Now check if $p = 2$

$2^3 + 3 = 8 + 3 = 11$, which is odd. So 2 is the only prime w/ this property.

1