2018 Mega Meet

Name:

Problem 1: Mental Math (no calculators allowed)

Example:

Question 1.1:

Question 1.2:

Question 1.3:

Question 1.4:

Question 1.5:

Question 1.6:

Question 1.7:

Question 1.8:

Question 1.9:

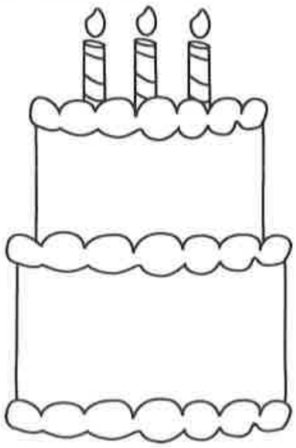
Question 1.10:

Name:

Problem 2: The Great Mathematical Baking Show

10 minutes, no calculators

You decide to bake a cake for your friend Ameen's birthday.



You have a lot of flour, but you only need 1 cup of flour for the cake. However, your kitchen is a mess and you are having trouble finding the right measuring tools. All you can find are a 5 cup measure and an 8 cup measure and a large bowl that holds plenty of flour. These measures have no markings on them, so you can only use them to measure 5 cups and 8 cups at a time. For example, you cannot use the 5 cup measure to scoop out 2 cups of flour.

Part 1:

You are nervous that you won't be able to get only 1 cup of flour, so you run to your friend Dorie, the baking and math expert, and ask for help.

"Don't worry! Using just the 5 cup and 8 cup measuring tools, we can get to 1 cup of flour!" Dorie tells you. She explains, "If you first add 8 cups of flour to the bowl, and then take away 5 cups of flour from the bowl, you will have 3 cups of flour left in the bowl. We can write that as an equation like this:"

[1] 8 cup scoop of flour + [-1] 5 cup scoop of flour = 3 cups of flour

"When we write a negative number, it means we remove that many scoops from the bowl. We didn't get 1 cup of flour, but we're closer. What if we add another 8 cups of flour to the bowl, and take away another 5 cups from the bowl?" Dorie suggests.

[2] 8 cup scoops of flour + [-2] 5 cup scoops of flour = 6 cups of flour

Dorie then suggests you organize your work in a table like this.

8 cup	5 cup	Bowl
1	-1	3
2	-2	6
		1

Question 2.1: (1 points) How many of each scoop do you need to get to 1 cup of flour?

8 cup scoops of flour:

5 cup scoops of flour:

With Dorie's help, you head back home and bake a delicious cake for Ameen's birthday.

The next day, you decide to bake another cake. Unfortunately, you forgot to clean up your kitchen and can only find the 12 cup measure and the 7 cup measure. First, let's try to get to 2 cups. The following table might help, but make sure you fill in your answer in the answer box.

12 cup	7 cup	Bowl
1	0	12
0	1	7
		14
		2

Question 2.2: (1 points) How can you get to 2 cups of flour?

12 cup scoops of flour:

7 cup scoops of flour:

Question 2.3: (2 points) How many of each scoop do you need to get to 1 cup of flour?

12 cup scoops of flour:

7 cup scoops of flour:

Part 2:

The next day, you decide to keep making cakes. But you can only find your 4 cup measure and your 10 cup measure. You are having trouble getting down to 1 cup of flour. You don't have very much time, so you want to make the smallest size of cake you can (obviously, you are going to make some sort of cake today!)

Question 2.4: (1 points) What is the smallest amount of flour you can possibly get in the bowl using a 4 cup measure and a 10 cup measure?

cups of flour

Question 2.5: (1 points) How many scoops of each measuring cup do you need to use to get the smallest amount of flour?

10 cup scoops of flour:

4 cup scoops of flour:

The next day, you can only find a 12 cup measure. You run to your neighbor Diego who will only let you borrow one of his measuring cups. Diego has an 18 cup measure, a 15 cup measure, and a 17 cup measure. You only have time to make 1 cake (remember that 1 cake needs exactly 1 cup of flour).

Question 2.6: (1 points) Which measuring cup should you borrow?

cup measure

Question 2.7: (1 points) How many scoops of each measuring cup do you need to get 1 cup of flour?

12 cup scoops of flour:

How many scoops from the other measuring cup?

For each of the following sets of measuring cups, what is the smallest number of cups of flour you can create?

Question 2.8: (1 points) A 15 cup measure and a 21 cup measure

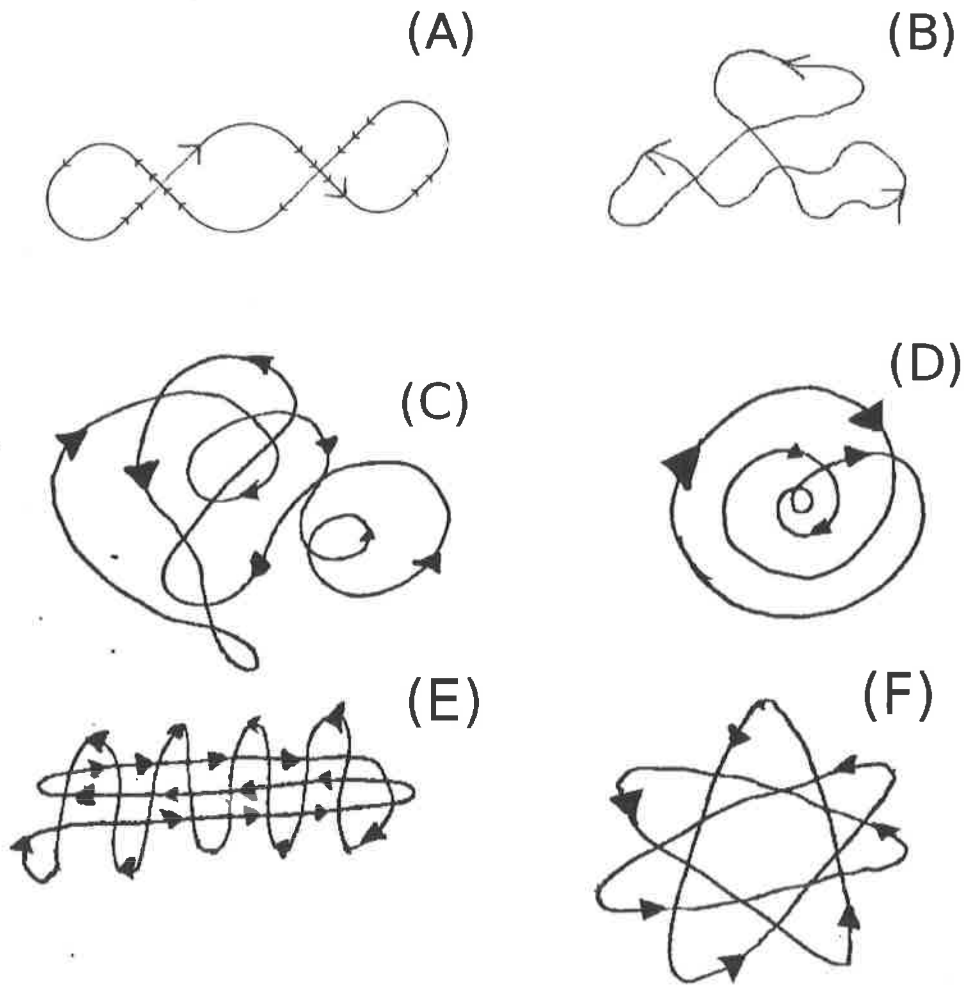
cups of flour

Question 2.9: (1 points) A 35 cup measure and a 50 cup measure

cups of flour

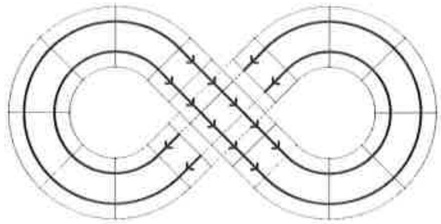
Part 4:

Wanlin has won millions of dollars in math competitions, and she has spent them all on new pieces for her racetracks. There's all sorts of curves and lines, but of course the separation between the two tracks is still 1 unit everywhere, so that all the pieces fit together. The tracks she is making now are so huge that we need to zoom way out to draw them, so far that it makes the track look like a line.



Question 6.7: (6 points) How much longer is Wanlin's route in each of the above tracks? (Beware that the answer could be a negative number if Wanlin's route is shorter!)

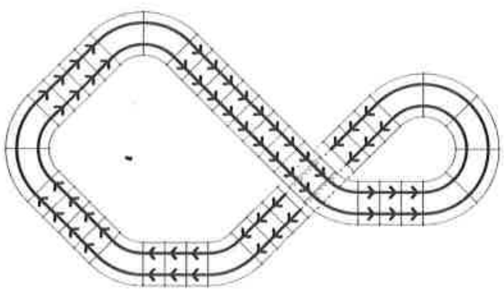
(A) :	units	(B) :	units
(C) :	units	(D) :	units
(E) :	units	(F) :	units



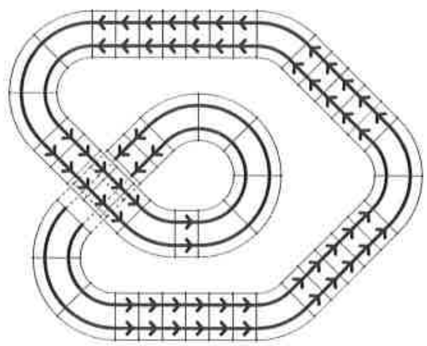
Question 6.5: (2 points) In the above track, how long is Wanlin's advantage on Louise?

units

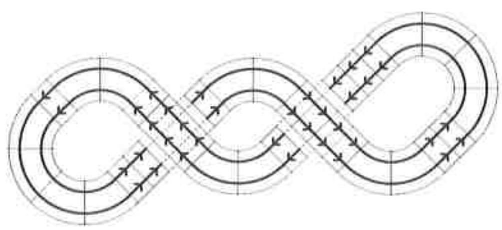
The bridges allow for building all sorts of new crazy tracks.



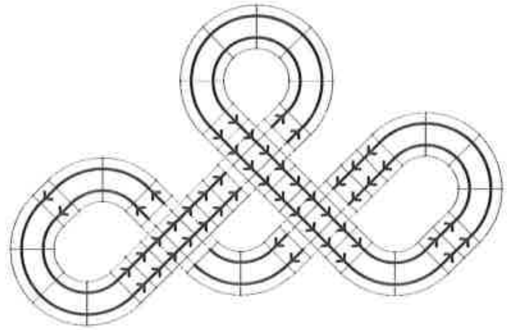
(A)



(B)



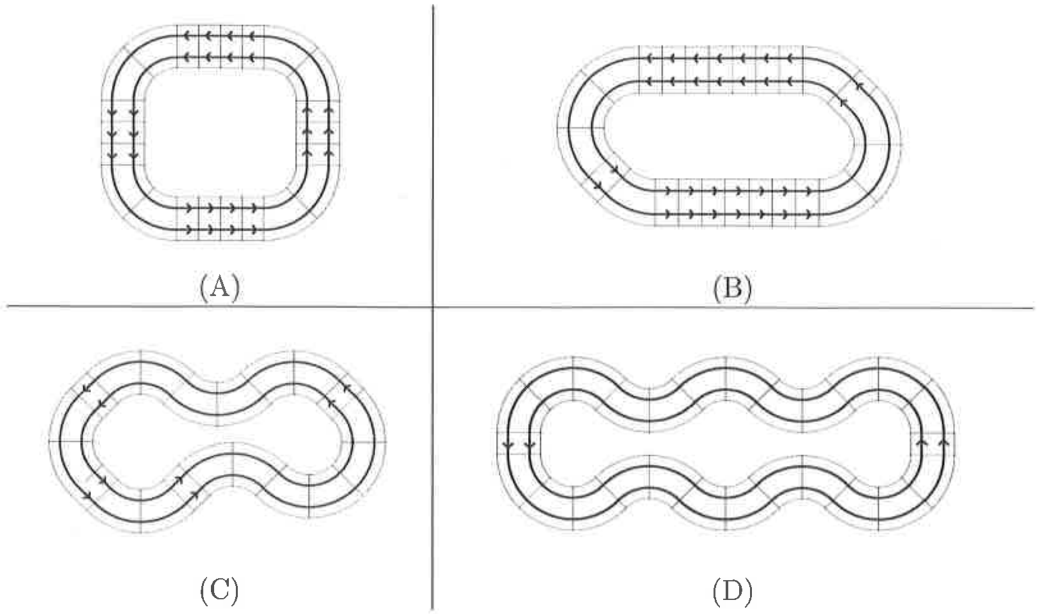
(C)



(D)

Question 6.6: (8 points) How much longer is Wanlin's route in each of the above tracks?

(A) :	units	(B) :	units
(C) :	units	(D) :	units



Question 6.3: (4 points) How much longer is Wanlin's route in each of the above tracks?

(A) :	units	(B) :	units
(C) :	units	(D) :	units

Question 6.4: (6 points) When they are not competing, they also enjoy trying to make different kinds of tracks. For the following sets of pieces, can they make a closed track using **all** the pieces in the set?

For each number of straight pieces, right turns and left turns, circle YES if it's possible to make a closed track using the pieces, or NO if it's impossible.

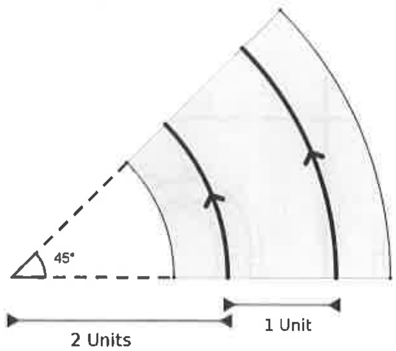
Straight pieces	Left turns	Right turns		Straight pieces	Left turns	Right turns	
4	8	0	YES / NO	0	5	3	YES / NO
3	8	0	YES / NO	0	12	8	YES / NO
4	20	16	YES / NO	500	508	500	YES / NO

Part 3:

Tired of always having to race on the outside route, Wanlin has figured out a way to mix things up, by building bridges that allow the track to cross itself. Now they can build these sorts of tracks:

Part 1:

Let's start by looking at the "left turn" piece of racetrack. Remember that the length of a circle of radius r is $2\pi r$!



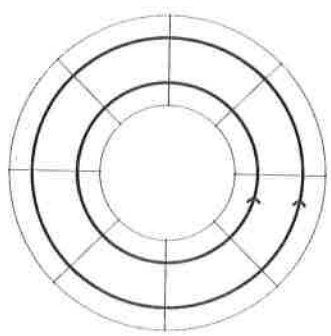
Question 6.1: (2 points) How long is Louise's (left) piece of the track?

units

How long is Wanlin's (right) piece of the track?

units

Now let's look at the circular track. Since Louise has the inside route, her path is shorter.



Question 6.2: (2 points) What is the difference between the lengths of Wanlin's path and Louise's? In other words, how much of a headstart should Louise give Wanlin?

units

Part 2:

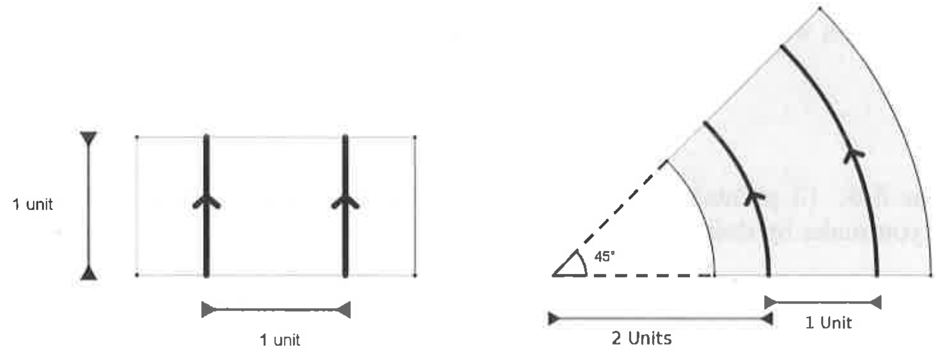
Louise and Wanlin have collected plenty of track pieces, so they can make all sorts of tracks, like these.

Team name:

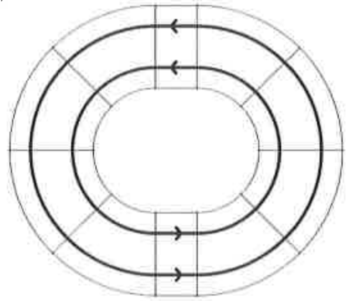
Problem 6: Louise and Wanlin fight over racetracks

30 minutes, no calculators

There's two things you should know about Wanlin and Louise: they like toy racetracks and they are extremely competitive. Their racetracks are made up of two kinds of pieces that they can assemble together into a closed track. These are the pieces.

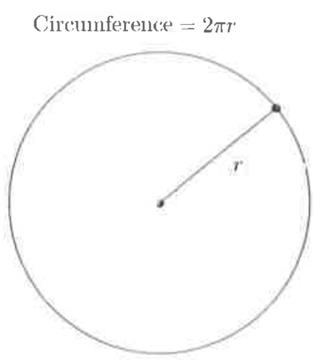


Louise likes to race her car on the **L**eft track, and Wanlin likes to use the **r**ight side. Being competitive as they are, they both know that the track is shorter if you take the inside route. For example, they can take 10 tracks and make them into this track:



Clearly Louise is running on a much shorter track! But the question is, how much of a headstart should Wanlin get so that they are both racing for the same distance?

For this problem, remember that a circle of radius r has length $2\pi r$. For all the answers that have a π in them, you can just leave π in the answer (no need to use 3.14's or anything of the sort).



Part 3:

After working so long with these braids, Brady noticed something curious: as words, the combinations AA', $A'A$, EE are different, but, as *braids*, they are the same (remember $AA' = A'A = EE = E$). As a first step to understanding this new mystery Brady came up with some new questions:

Question 5.7: (2 points) Using the letters A, A' and E, how many different words of two letters can you write?

Question 5.8: (3 points) Using the movements A, A' and E, how many different braids can you make by doing two movements?

With these answers, Brady now feels that he can have a better understanding of the whole picture.

Question 5.9: (2 points) Using the letters A, A', B, B' and E, how many words of two letters can you write?

Question 5.10: (5 points) Using the movements A, A', B, B' and E, how many different braids can you make by doing two movements?

After practicing with just two movements, Brady wants to go further.

Question 5.11: (2 points) Using the letters A, A', B, B' and E, how many words of three letters can you write?

Question 5.12: (3 points) Using the movements A, A', B, B' and E, how many different braids can you make by doing three movements?

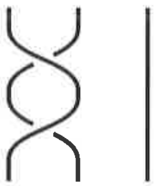
Brady was about to start experimenting with four movement braids but his dad called him for dinner. He'd have to return to this question another time.

What is Brady's name for this section of the "usual" braid?

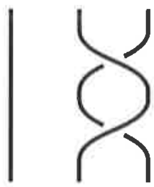
Part 2:

Brady discovered that sometimes he can move the strands in the middle of a braid into different positions without moving the ends. He considers these to be the same braid, so there are many examples where the same braid might have different names. For example, the braids AA' and BB' are the same as E.

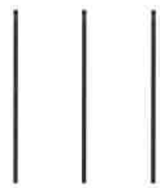
AA'



BB'



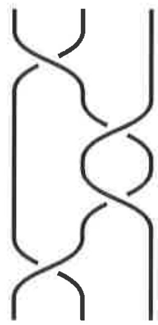
E



When we can follow one braid by another to form something equivalent to the braid E, we say that the second braid is the *inverse* of the first. For example, the drawings above show that A' is the inverse of A and B' is the inverse of B.

As another example, the inverse of AB' is BA'. As you can see in the following picture, the braid $AB'BA'$ can have the strands moved in the middle to form the braid E:

$$AB'BA' = E$$



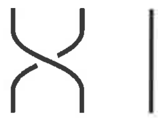
Question 5.4: (1 points) What braid is the inverse of A'?

Question 5.5: (2 points) What braid is the inverse of ABA'?

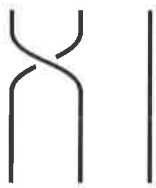
Question 5.6: (4 points) Brady has already braided his hair with the braid $A'BA'B$, but he's decided that he really wants to have the braid $A'BBAB'$ in his hair. What braid should he follow $A'BA'B$ with in order to produce the braid $A'BBAB$?

Finally, Brady only cares about how the strands are crossed, not how long they are, which means, for example, that the braids A, AE, and EA are all considered the same and we can write $A = AE = EA$:

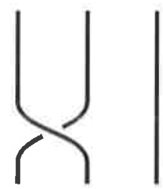
A



AE

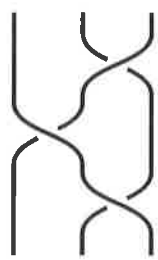


EA

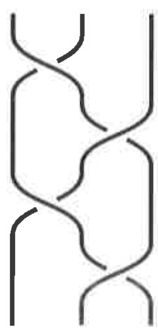


Question 5.1: (2 points) Draw the braid that Brady would call $BBA'B'$.

Question 5.2: (2 points) Using Brady's notation, how would you write the name of the following braid?



Question 5.3: (2 points) Here is a section of the "usual" braid.



Team name:

Problem 5: Brady's Braids

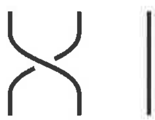
30 minutes, no calculators

Brady loves braiding hair, and recently he started thinking about all the different patterns he could make. As he enjoys thinking mathematically, he decided to write down some names for the different braiding operations and see if he could study the rules of this game.

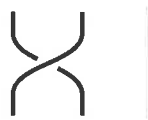
Part 1:

Usually, Brady separates his hair into just three sections and braids these together. There are four basic steps he uses, so he calls them A, A', B, and B'. They are drawn below:

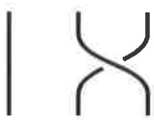
A



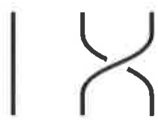
A'



B



B'

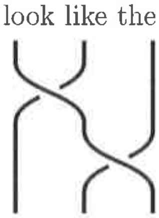


Note: The braid A differs from A' because the left strand is passed over the right strand in A, but the left strand is passed under the right strand in A'. B and B' differ from each other in the same way.

Brady uses the letter E for a step where none of the strands are passed over each other. In other words, the braid E looks like this:

E

To create more complicated braids, Brady performs those basic steps in sequence from top to bottom. For example, first doing the braid A, and then following with the braid B, which we will write as AB, would look like the following picture:



Question 4.6: (1 points) What shape is dual to the cube?

Question 4.7: (1 points) What shape is dual to the octahedron?

Part 3: Structure of the dodecahedron and icosahedron

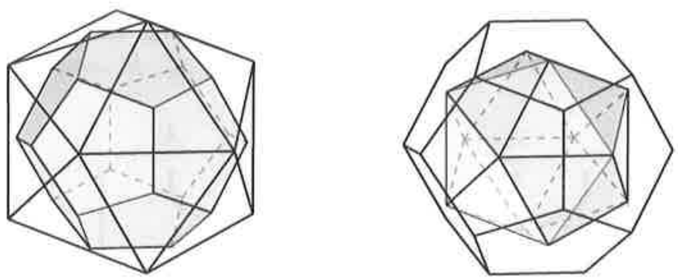


FIGURE 5. On the left, a dodecahedron inside of an icosahedron. On the right, an icosahedron inside of a dodecahedron

Here is another image of a pair of dual polyhedra. This time, the shapes are the *dodecahedron* and the *icosahedron*. These names come from the Greek words for 'twelve' and 'twenty', because the dodecahedron is made of 12 pentagonal faces, and the icosahedron is made of 20 triangular faces.

Question 4.8: (1 points) How many vertices and edges does the regular dodecahedron have?

vertices	edges

Question 4.9: (1 points) How many vertices and edges does the regular icosahedron have?

vertices	edges

Here's the construction of a mysterious, magical solid: the stellation of the icosahedron. We take an icosahedron, and on **each** of its 20 triangular faces, we attach a tetrahedron. The result is referred by expert polyhedrologists as the **super-amazingly-awesome-starry-pointy-hedron**.

Question 4.10: (1 points) How many vertices, edges and faces does the **super-amazingly-awesome-starry-pointy-hedron** have?

vertices	edges	faces

FIGURE 3. Euler's face is worth 10 Francs.

We can start by trying to put our data in a table. We left some blank spaces so you can fill in with other examples of solids to see if you can find the pattern. It turns out that if we combine those numbers for each shape in a particular way, we always get 2. But what is the way?

Solid	Tetrahedron	Cube	Octahedron				
Vertices	4	8					
Edges	6	12					
Faces	4	6					

Question 4.4: (2 points) If V is the number of vertices in one of these solids, E is the number of edges, and F is the number of faces, what numbers complete the following formula?

$$V + \_\_\_E + \_\_\_F = 2$$

Part 2: Dual Polyhedra

Some pairs of polyhedra are said to be *dual* to each other, relating to each other in a particular way. Look at the following pictures:

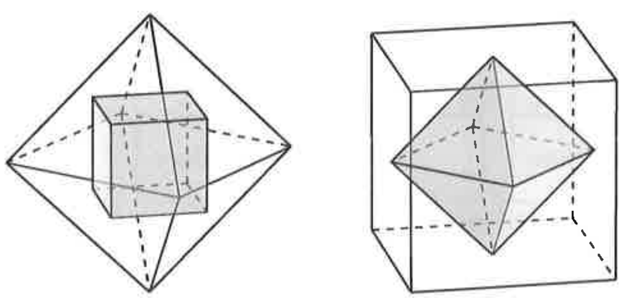


FIGURE 4. A cube and octahedron superimposed.

As you can see, the cube and octahedron are related to each other geometrically. To obtain the dual of a polyhedron, we turn every vertex into a face and every face into a vertex, connecting the new vertices with new edges that cross the old edges.

Question 4.5: (1 points) What shape is dual to the tetrahedron?

Question 4.1: (1 points) How many vertices, edges, and faces does the octahedron have?

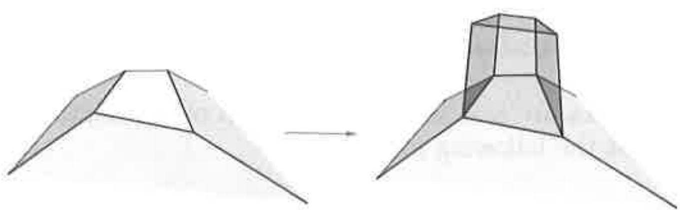
vertices	edges	faces

Part 1: Patterns in the numbers of vertices, edges and faces

First of all, if we're going to figure out a pattern, we should look for ways to come up with new polyhedra. For example, whenever we have a polyhedron with a triangular face, we can attach a tetrahedron to this triangle to make a new polyhedron, like so:

The tetrahedron has 4 vertices, 6 edges and 4 faces. However, we notice that when we attach the tetrahedron, the original triangle is no longer a face of the polyhedron, and not all the vertices and edges are truly new, since some belonged to the original triangle. As a result, it turns out that **if we attach a tetrahedron to a triangular face, the net change is that we have added 1 vertex, 3 edges and 2 faces.**

What happens if we attach other polyhedra? We can attach a cube to a square face:



Question 4.2: (1 points) What is the **net change** in the number of vertices, edges and faces when we attach a cube to a square face?

vertices	edges	faces

We can attach all sorts of things to a polyhedron to make new ones! How about attaching an octahedron? (We've already seen a picture of one!)

Question 4.3: (1 points) What is the **net change** in the number of vertices, edges and faces when we attach an octahedron to a triangular face of a polyhedron?

vertices	edges	faces

You might have noticed a pattern relating the number of vertices, edges, and faces a solid has. Leonhard Euler also noticed the pattern and for that and many other things his face is now on Swiss money.

Name:

Problem 4: The Many-Headed Polyhedra

15 minutes, no calculators

We are looking at solids, known as polyhedra to fancy people. These are three-dimensional shapes that you can make by joining polygons by their edges until you have something solid! Polygons are flat shapes made of straight edges, like triangles and squares. You already know some solids. For example, the triangular pyramid (also 'tetrahedron'), and the cube.

FIGURE 1. A triangular pyramid (tetrahedron) and a cube

There are many things we might want to know about these. Maybe the simplest is the number of *vertices* ("corners"), *edges*, and *faces* each has.

From the pictures of the tetrahedron and the cube, maybe together with a bit of imagination, we can count the numbers of vertices, edges and faces on them: the tetrahedron has 4 vertices, 6 edges, and 4 faces; the cube has 8 vertices, 12 edges, and 6 faces.

Let's look at a more complicated solid, the octahedron:

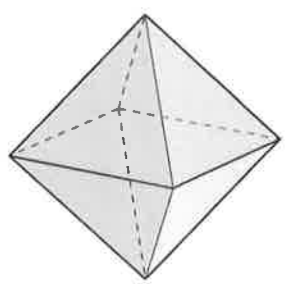


FIGURE 2. An octahedron

Question 3.7: (1 points) What is the **smallest** number of diagonals that an irregular pentagon can possibly have?

diagonals

Question 3.8: (1 points) What is the **smallest** number of diagonals that an irregular hexagon can possibly have?

diagonals

Question 3.9: (2 points) What is the **smallest** number of diagonals that an irregular n-sided polygon can possibly have?

diagonals

Number of sides	Number of diagonals through one vertex	Total number of diagonals
4	1	2
5	2	5
6		
7		
20		
n		

Question 3.5: (1 points) How many diagonals does a regular 20-sided polygon have?

diagonals

Part 2:

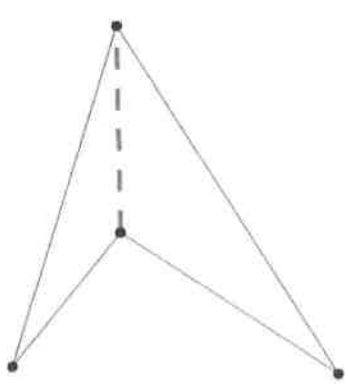
Let's mix it up a little bit now.

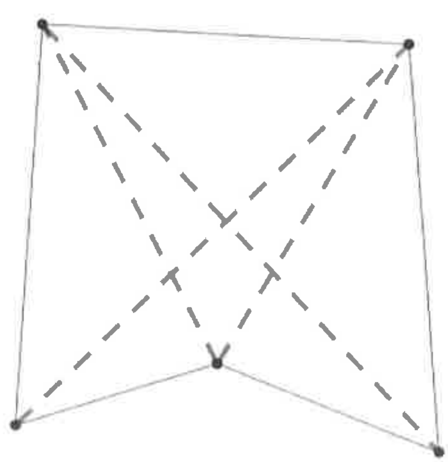
Question 3.6: (1 points) Now say everyone in your team shakes hands with everyone else. If there are 8 people in your team, what is the total number of handshakes that take place?

handshakes

Part 3:

Finally, we will look at all possible polygons and not just the regular ones. For example, the quadrilateral below has just one diagonal instead of two, the pentagon in the middle has 4 diagonals instead of 5 and the pentagon on the right just has 2 diagonals.





Part 1:

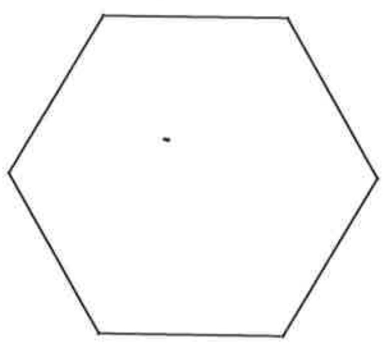
Question 3.1: (1 points) Below is a regular hexagon. How many diagonals does it have? (You're free to draw them all, but make sure to write the number in the answer box!)

diagonals

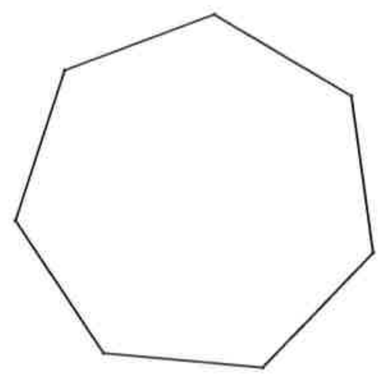
Question 3.2: (1 points) How about a regular heptagon? (A heptagon is a 7 sided polygon.)

diagonals

Hexagon:



Heptagon:



We would really like to know how many diagonals are in something like a 1,000,000-sided polygon, so we want to try a different strategy to count the diagonals without having to draw them. Let's try in different steps.

Question 3.3: (1 points) Take one of the vertices in a regular 10-sided polygon. How many diagonals have this vertex as one of its endpoints?

diagonals

Question 3.4: (1 points) Take one of the vertices in a regular n-sided polygon. How many diagonals have this vertex as one of its endpoints?

diagonals

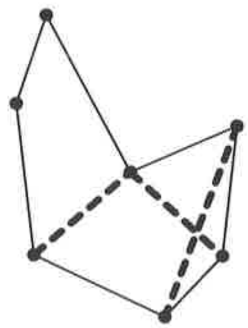
For the next questions, feel free to fill in this table to try to find the pattern, or not, as you wish!

Name:

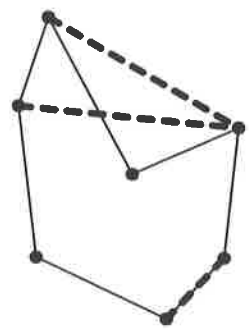
Problem 3: Counting diagonals

12 minutes, no calculators

We are counting diagonals! What are diagonals, you say? Diagonals are segments joining two vertices in a polygon. Diagonals are always inside the polygon, don't let them fool you.



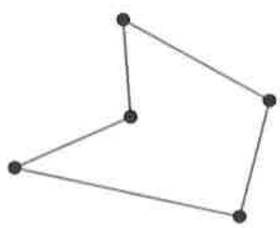
The dotted lines are diagonals



The dotted lines are not diagonals

For some of the problems, we will only be looking at regular polygons, which are polygons that look the same after we rotate them.

This pentagon is regular.



This pentagon is not regular.

This is because nonregular polygons are more complicated, as some have diagonals that don't exist because they are outside of the polygon!

For example. A regular pentagon has 5 diagonals, as you can see.